PMHT for Tracking with Timing Uncertainty

Brian Cheung†,⋆ brian.cheung@dsto.defence.gov.au
Samuel Davey† samuel.davey@dsto.defence.gov.au
Douglas Gray⋆ dgray@eleceng.adelaide.edu.au
†Defence Science and Technology Organisation, Australia
⋆School of Electrical and Electronic Engineering, The University of Adelaide, Australia

Abstract – *This paper considers the problem of tracking a target using measurements where the temporal information is noisy or unreliable. The measurements are modelled as being received with a stochastic time delay. By treating the true measurement time as missing data and using Expectation-Maximisation, a modified version of the Probabilistic Multi Hypothesis Tracker (PMHT) is derived for the uncertain timing problem. This modified PMHT thus associates each measurement to the targets and to time instants. The method is demonstrated using simulations and the improvement quantified over using the standard PMHT in ignorance of the noisy time data.*

Keywords: Tracking, probabilistic multi-hypothesis tracker (PMHT), time uncertainty.

1 Introduction

The role of a tracking filter is to perform recursive target state estimation given a set of noisy position measurements collected at known times. It is customary to assume that there is no error in the timing information, and this assumption is reasonable for the case of a single electro-magnetic sensor. In such a case, the propagation delay from sensor to target and back is negligible and the tracker is situated at the sensor, so it may know the sensor clock. However, for other situations, such as distributed sensing, the timing information may not be so reliable. As the trend for sensor networks heads towards the usage of low cost sensors such as motes and other low bandwidth devices, this time uncertainty error will need to be accounted for.

Relatively little work has been done in the area of estimation with uncertain timing information. Li and Leung considered a fixed timing bias in the context of sensor registration [1]. Their algorithm used Expectation-Maximisation (EM) [2] to estimate time, range and azimuthal bias parameters. Morelande [3] considered a problem where some measurements have accurate time information, but others do not. He used the accurate time information to estimate the parameters of the unreliable time-stamps via a particle filter. Orguner and Gustafsson [4, 5] considered a problem where the propagation delay was significant and thus incurred timing errors that were a function of the target state. Both of the tracking methods above extended the state vector in order to estimate the delay and hence improve tracking.

This paper uses EM, in the guise of the Probabilistic Multi-Hypothesis Tracker (PMHT) of Streit and Luginbuhl [6], to estimate target trajectories when the measurement time is not known reliably. The PMHT is a data association algorithm derived from the application of the EM algorithm to target tracking. The PMHT uses EM to model the assignment of measurements to targets as hidden variables and estimates target states by taking the expectation over the assignments. The advantage of the PMHT over other conventional data association techniques is that it has linear complexity in the number of targets, the number of measurements per frame, and in the number of frames. In contrast, other data association methods incur a complexity that grows with the number of permutations of measurements and targets within a frame and then exponentially with time.

The PMHT performs data association by introducing an assignment index that is an unobserved variable and then marginalising over it via EM. This paper extends this model by introducing another assignment index that identifies the true time at which a measurement was collected. Again, this index is unobserved and the EM method is used to marginalise over it. The result is a set of weights that probabilistically associate each measurement with both tracks and time slots. It is shown that the resulting EM auxiliary function is equivalent to a problem with known observation times and known measurement origin using synthetic measurements and a modified measurement function.

2 Problem Formulation

Assume that there is a sensor that is tracking M targets. At discrete scans, it collects observations, some of which are due to the targets, and some of which are false alarms, referred to as clutter. Let the total number of observations be N and the number of scans be T.

Let the state of target m at time t be denoted x_t^m, and let

X^m denote the set of all states for target m, i.e. $X^m = \{x_t^m\}$ for $t = 1 \ldots T$. Similarly, let the set of all states across the targets be denoted by $X = \{X^m\}$.

It is assumed that the prior distribution of the state of each target is known and is given by $\psi_0^m(x_0^m)$ for target m. The target dynamics are also assumed to be known and can be described by the evolution probability density function (pdf) $\psi_t^m(x_t^m|x_{t-1}^m)$.

Let the nth measurement received by the sensor be denoted by z_n. Each measurement consists of a state component, z_n^x, and a timestamp, z_n^τ, both of which are noisy observations.

Let the true source of measurement n be $k_n \in 0 \ldots M$, where $k_n = 0$ means that the measurement is a false alarm and $k_n = m$ implies that measurement n is an observation of target m. Let the true collection time of measurement n be $\tau_n \in 1 \ldots T$. Both of these are, of course, unknown and are treated as random variables with priors π_{nm}^k and π_{nt}^τ respectively. These prior distributions are also unknown.

Let Z denote the set of all measurements, K denote the set of all measurement-to-platform assignments and τ denote the set of all measurement-to-time assignments. Similarly, Π^k and Π^τ denote the collection of track and time assignment priors respectively.

The sensor observation process is described by a known measurement pdf that is denoted as $\zeta^x(z_n^x|x_t^m)$, where $m = k_n$ and $t = \tau_n$. In general PMHT is capable of handling a time and target dependent measurement function, but here we choose to assume a constant one for notational clarity.

Similarly, the timing error has a known probability mass function (pmf) that is denoted as $\zeta^\tau(z_n^\tau|\tau_n)$

3 PMHT for tracking with timing uncertainty

PMHT is based on the application of EM to multi-target data association. The major advantage of using PMHT is that the computational complexity increases linearly with the number of targets, measurements and time steps unlike other association algorithms which can grow exponentially. This allows the algorithm to be implemented without approximation and allows for efficient smoothing over time batches when the application requires.

The PMHT algorithm is a method for finding the best estimate of the target states, X, when the measurement source K is unknown. It does this by treating the assignments as missing data using EM. The state estimate is derived iteratively by maximising an auxiliary function

$$Q\left(X|\hat{X}(i)\right) = \sum_K P\left(K|\hat{X}(i), Z\right) \log P(X, K, Z), \quad (1)$$

where i is an iteration index. Upon convergence, the algorithm's output is the state estimate. This auxiliary function $Q(.)$ can be maximised using any appropriate estimator. It can be shown that the auxiliary function is equivalent to the log-likelihood of a known assignment problem with synthetic measurements determined by the expectation step [6]. Thus for linear Gaussian cases, the Kalman filter may be used to solve the equivalent problem. For nonlinear problems such as tracking with range and bearing measurements, a nonlinear filter must be used, such as an Extended Kalman Filter or a particle filter.

For clarity, the abbreviation PMHT will be used to refer to the standard PMHT algorithm, i.e. tracking assuming perfect timing information. The abbreviation PMHT-t will be used to refer to the modified PMHT that is capable of compensating for unreliable timing information. The derivation of the PMHT-t is similar to the standard PMHT derivation presented in detail in [6] and [7]; the difference is the inclusion of the extra hidden variable τ and the extended measurement function $\zeta^\tau(z_n^\tau|\tau_n)$.

In EM terminology, the complete data are (X, τ, K, Z), the incomplete data are (X, Z), and (τ, K) are the missing data. The auxiliary function is the expectation of the complete data log-likelihood over the missing data, which now takes the form:

$$\begin{aligned} &Q(X, \Pi^\tau, \Pi^k|\hat{X}(i), \hat{\Pi}^\tau(i), \hat{\Pi}^k(i)) \\ &\quad = \sum_K \sum_\tau P(\tau, K|\hat{X}(i), Z) \log P(X, \tau, K, Z), \end{aligned} \quad (2)$$

where the summation is over all permutations of the assignment variables τ and K.

For compactness, let

$$\sum_{n,t,m} (\cdot) \equiv \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{M} (\cdot), \quad (3)$$

i.e. a sum over all of the measurements at each time and from each sensor.

Due to the independence assumptions, the complete data likelihood becomes:

$$P(X, \tau, K, Z) = P(X) P(\tau; \Pi^\tau) P(K; \Pi^k) P(Z|X, \tau, K), \quad (4)$$

where

$$P(X) = \prod_{m=1}^{M} \psi_0^m(x_0^m) \prod_{t=1}^{T} \psi_t^m(x_t^m|x_{t-1}^m), \quad (5)$$

$$P(\tau; \Pi^\tau) = \prod_{n=1}^{N} \pi_{n\tau_n}^\tau, \quad (6)$$

$$P(K; \Pi^k) = \prod_{n=1}^{N} \pi_{nk_n}^k, \quad (7)$$

$$P(Z|X, \tau, K) = \prod_{n=1}^{N} \zeta^x\left(z_n^x|x_{\tau_n}^{k_n}\right) \zeta^\tau\left(z_n^\tau|\tau_n\right) \quad (8)$$

The conditional probability of the missing data,

$P(\tau, K|X, Z)$, can be determined using Bayes' Rule:

$$\begin{aligned}
&P(\tau, K|X, Z)\\
&= \frac{P(X, \tau, K, Z)}{\sum_{\tau', K'} P(X, \tau, K', Z')}\\
&= \frac{P(X)P(\tau)P(K)P(Z|X, \tau, K)}{P(X) \sum_{\tau', K'} P(\tau')P(K')P(Z|X, \tau', K')}\\
&= \frac{\prod_n \pi^{\tau}_{n\tau_n} \pi^{k}_{nk_n} \zeta^x(z_n|x^{k_n}_{\tau_n})}{\sum_{\tau', K'} \prod_n \pi^{\tau}_{n\tau'_n} \pi^{k}_{nk'_n} \zeta^x(z_n|x^{k'_n}_{\tau'_n})}\\
&= \prod_n \frac{\pi^{\tau}_{n\tau_n} \pi^{k}_{nk_n} \zeta^x(z^x_n|x^{k_n}_{\tau_n}) \zeta^{\tau}(z^{\tau}_n|\tau_n)}{\sum_{t=1}^{T} \sum_{m=0}^{M} \pi^{\tau}_{nt} \pi^{k}_{nm} \zeta^x(z^x_n|x^m_t) \zeta^{\tau}(z^{\tau}_n|t)}\\
&\equiv \prod_n w_{n\tau_n k_n} \qquad (9)
\end{aligned}$$

Thus the conditional probability of the assignments is given by the product of individual per measurement *weights*. Each weight, w_{ntm}, is the normalised likelihood of the nth measurement from target m at time t. The numerator of the weight is simply the product of the assignment priors, the positional measurement likelihood and the temporal measurement likelihood.

Combining the two equations (4) and (9) leads to the auxiliary function to be maximised:

$$\begin{aligned}
&Q(X, \Pi^{\tau}, \Pi^{k}|\hat{X}(i), \hat{\Pi}^{\tau}(i), \hat{\Pi}^{k}(i))\\
&= \log P(X) + \sum_{n,t,m} w_{ntm} \log \pi^{\tau}_{nt} + \sum_{n,t,m} w_{ntm} \log \pi^{k}_{nm}\\
&+ \sum_{n,t,m} w_{ntm} \log \zeta^x(z^x_n|x^m_t) + \sum_{n,t,m} w_{ntm} \log \zeta^{\tau}(z^{\tau}_n|t)\\
&\equiv Q_X + Q^{\tau}_{\Pi} + Q^{k}_{\Pi} + Q_{\tau} \qquad (10)
\end{aligned}$$

The term Q_τ in (10) is given by

$$Q_{\tau} = \sum_{n,t,m} w_{ntm} \log \zeta^{\tau}(z^{\tau}_n|t)$$

and is constant so it has no influence on the optimisation.

The term Q^{τ}_{Π} in (10) is given by

$$Q^{\tau}_{\Pi} \equiv \sum_{n,t,m} w_{ntm} \log \pi^{\tau}_{nt},$$

and is similar to that of the standard multi-sensor PMHT [8]. It is maximised subject to the constraint that $\sum_t \pi^{\tau}_{nt} = 1$ using a Lagrangian, resulting in the updated prior estimate

$$\hat{\pi}^{\tau}_{nt}(i+1) = \sum_{m=1}^{M} w_{ntm}, \qquad (11)$$

i.e. the weights' relative frequency for time t.

Similarly, the Q^{k}_{Π} term results in a relative frequency estimate for the locale to platform assignment prior

$$\hat{\pi}^{k}_{nm}(i+1) = \sum_{t=1}^{T} w_{ntm}. \qquad (12)$$

The remaining term, Q_X, couples the target states and the measurements and is given by

$$Q_X \equiv \log P(X) + \sum_{n,t,m} w_{ntm} \log \zeta^x(z_n|x^m_t). \qquad (13)$$

If the measurement function is Gaussian, it can be shown that this function is equivalent to the log likelihood of a tracking problem with known data association [9]. Intuitively, if $\zeta^x(z_n|x^m_t)$ is Gaussian, then the summation in (13) is a linear combination of quadratics, which is itself a quadratic. This allows us to write (13) as

$$Q_X \equiv \log P(X) + \sum_{t,m} \log \tilde{\zeta}^x_t(\tilde{z}^m_t|x^m_t), \qquad (14)$$

where the synthetic measurement, $\tilde{z}^m_t$, is given by

$$\tilde{z}^m_t = \frac{1}{\sum_{n=1}^{N} w_{ntm}} \sum_{n=1}^{N} w_{ntm} z_n, \qquad (15)$$

and the synthetic measurement function, $\tilde{\zeta}(\cdot)$, is a Gaussian pdf with the same mean as the true measurement function and a variance that is a scaled version of the sensor measurement variance, R,

$$\tilde{R}^m_t = \frac{1}{\sum_{n=1}^{N} w_{ntm}} R. \qquad (16)$$

The track for target m is now refined by smoothing the synthetic measurements and covariances.

The PMHT-t consists of iteratively calculating assignment weights, w_{ntm}, and estimating the target tracks and assignment priors until convergence. For a linear Gaussian problem this amounts to:

1. Initialise target state estimates, measurement-to-track assignment priors and measurement-to-time assignment priors.
2. Calculate the assignment weight for each measurement and track at each scan, according to (9).
3. Update the assignment priors using (11) and (12).
4. Determine synthetic measurements and covariances for each target at each scan using (15) and (16).
5. Update the target state estimates using a Kalman smoothing algorithm over the synthetic measurements and covariances.
6. repeat steps 2 to 5 until convergence.

4 Performance Analysis

The use of PMHT for tracking with time uncertainty is now illustrated through simulation. In the simulations, we assumed there a Cartesian sensor observing targets in the plane. The moving targets had a state vector consisting of 2-D position and velocity. For each scenario, $T = 100$ scans were simulated.

The target states were assumed to follow an almost-constant-velocity model independently in X and Y. Thus the state evolution process was defined by

$$\psi_t^m(x_t^m | x_{t-1}^m) = \mathcal{N}\left(x_t^m; F x_t^m, Q\right), \tag{17}$$

where $\mathcal{N}(t; \mu, \Sigma)$ is a multivariate Gaussian,

$$F = \begin{bmatrix} F_2 & 0 \\ 0 & F_2 \end{bmatrix} \quad \text{with} \quad F_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

and

$$Q = 10^{-8} \begin{bmatrix} Q_2 & 0 \\ 0 & Q_2 \end{bmatrix} \quad \text{with} \quad Q_2 = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}.$$

The positional measurement function of the sensor was linear and Gaussian,

$$\zeta_t^m(z_n | x_t^m) = \mathcal{N}\left(z_n; H x_t^m, R\right), \tag{18}$$

with $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $R = 10^{-2} I$.

The time measurement model of the sensor was a stochastic delay: a measurement cannot be received before the true collection time, i.e. $\tau_n \leq z_n^\tau$, and the delay probability decays exponentially:

$$\zeta^\tau(z_n^\tau | \tau_n) = \begin{cases} 0 & z_n^\tau < \tau_n, \\ p(1-p)^{(z_n^\tau - \tau_n)} & z_n^\tau \geq \tau_n. \end{cases} \tag{19}$$

Thus zero delay occurs with probability p and the mean delay is $(1-p)/p$.

The sensor was assumed to have a constant probability of detection, P_d. The number of false alarms was Poisson distributed with mean ρ false alarms per scan, and the false alarms with uniformly distributed in space.

The measurement-to-track assignment prior was initialised with

$$\hat{\pi}_{nm}^x(0) = \begin{cases} \rho + \epsilon & m = 0, \\ P_d & m > 0, \end{cases} \tag{20}$$

where ϵ is a small constant used to ensure good conditioning for very low clutter densities. This initialisation is not normalised, but due to the ratio in the weight calculation (9), the scale factor cancels out and has no effect.

A sliding batch with $T = 10$ was run over the measurements and the measurement-to-time assignment prior was initialised with

$$\hat{\pi}_{nt}^\tau = \frac{1}{T}. \tag{21}$$

Two measures of performance were used to quantify the quality of the track output. Firstly, the fraction of misassociated tracks was determined, where a missassociated track was defined as one that did not remain within a prescribed circular gate of the true target location for the whole scenario duration. For example, a track that swaps from one target to another was classed as misassociated, or one that diverged from the target path due to clutter. Secondly, the 2-D RMS position and velocity estimation errors were determined. Misassociated tracks were not included in the error calculation.

Two scenarios are presented here. In the first scenario, there were four targets travelling with constant-velocity motion as illustrated in figure 1. Figure 1(a) shows the measurements that the targets produce with a P_d of 0.9 and no clutter ($\rho = 0$). The coloured boxes show the target initial locations. Two of the targets had clear paths, not close to the others, but two remaining two targets crossed part way through the scenario.

Figures 1(b) and (c) show the tracks resulting from running the PMHT-t and the standard PMHT respectively. In this particular example, due to the timing uncertainty, the standard PMHT has misassociated the crossing targets whereas the PMHT-t has tracked the targets correctly.

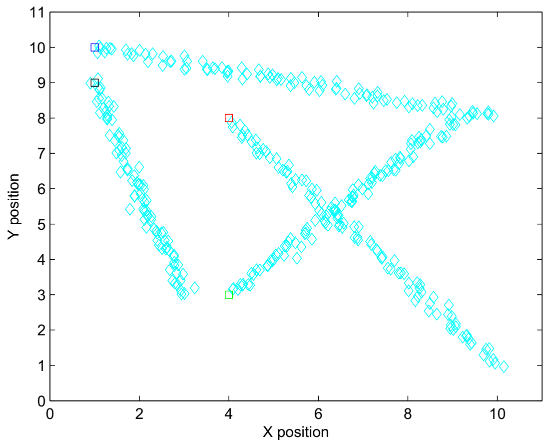
The second simulation scenario involved a single target performing a manoeuvre. The target initially travelled East with constant velocity before performing a coordinated turn manoeuvre with constant turn rate and then travelling West, again with constant velocity. Figure 2a shows the measurements that this target trajectory produces with a P_d of 0.9 and no clutter ($\rho = 0$).

Figures 2(b) and (c) show the results from running PMHT-t and the standard PMHT respectively. In this particular case, the standard PMHT has not been able to follow the manoeuvre due to the time errors and has broken track whereas the PMHT-t has been able to track the target.

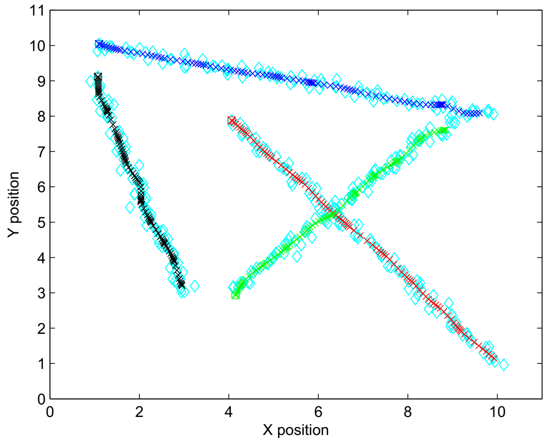
Monte Carlo simulations of the above scenarios were performed, and the number of misassociated tracks and RMS in position and velocity was averaged over 100 trials for different timing delay parameters, p. The results for scenario one and two are in Table 1 and Table 2 respectively. In order to test the sensitivity of the PMHT-t to the mismatch in the timing delay pmf, two versions of PMHT-t were tested. In one, a fixed value of $p' = 0.7$ was used, while the true p used to generate data was varied. In the second, the PMHT-t was provided with the true parameter value.

Both table 1 and table 2 show that in the case where $p = 1$, the PMHT and the PMHT-t give the same results. In this case, there is no timing error. As the value of p decreases, there is a higher probability of time delay error in the measurements and the targets are more difficult to track. As evident in Table 1, the RMS error for the standard PMHT is slightly greater than that of the PMHT-t. There are also less misassociated tracks with the PMHT-t.

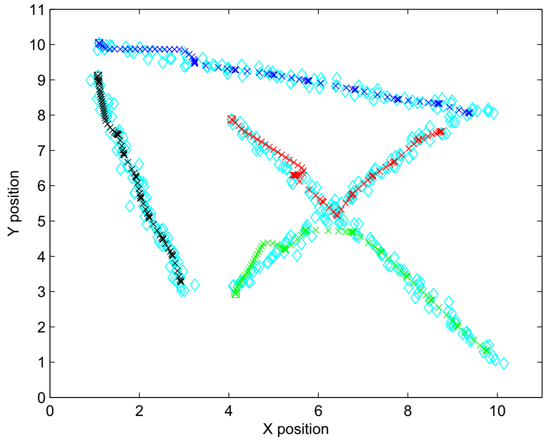
For the straight-line scenario, there was very little difference in performance. This was not the case for the turn scenario, as shown in Table 2. In this scenario, when there was



(a) Simulated measurements

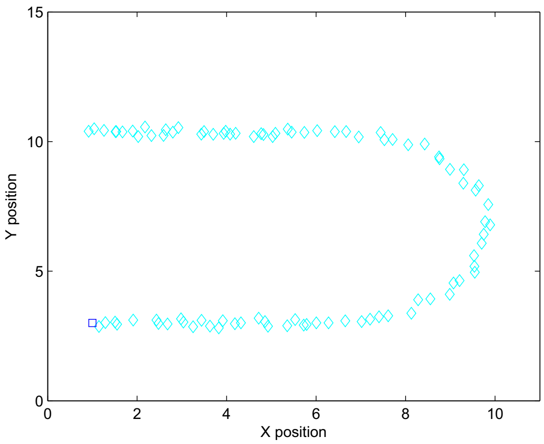


(b) PMHT-t results

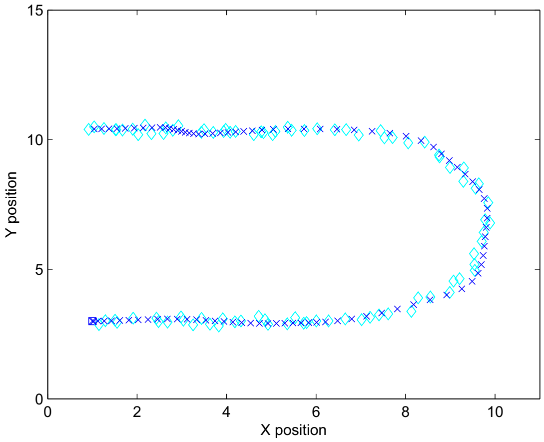


(c) PMHT results

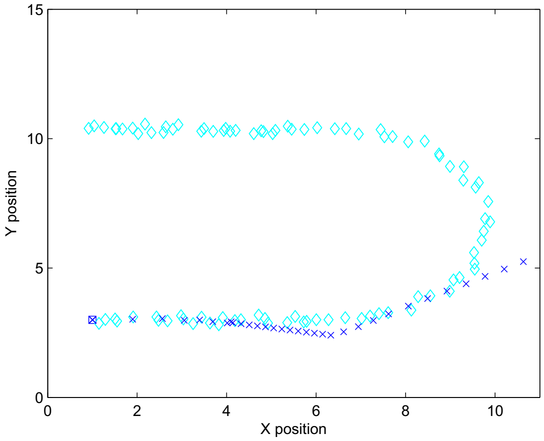
Figure 1: Scenario 1 Tracking Results



(a) Simulated measurements



(b) PMHT-t results



(c) PMHT results

Figure 2: Scenario 2 Tracking Results

Table 1: Scenario 1 Monte Carlo RMS results

p	1.0	0.7	0.5	0.3
av. delay	0	0.4	1	2.3
Position RMS				
PMHT	0.09	0.21	0.16	0.27
PMHT-t	0.09	0.11	0.14	0.23
PMHT-t ($p = 0.7$)	0.09	0.11	0.14	0.23
Velocity RMS				
PMHT	0.02	0.03	0.04	0.07
PMHT-t	0.02	0.03	0.03	0.05
PMHT-t ($p = 0.7$)	0.02	0.03	0.03	0.05
Misassociated				
PMHT	0	0.01	0.03	0.06
PMHT-t	0	0.01	0.02	0.04
PMHT-t ($p = 0.7$)	0	0.01	0.02	0.04

Table 2: Scenario 2 Monte Carlo RMS results

p	1.0	0.7	0.5	0.3
av. delay	0	0.4	1	2.3
Position RMS				
PMHT	0.34	0.73	0.86	1.18
PMHT-t	0.34	0.24	0.35	0.57
PMHT-t ($p = 0.7$)	0.17	0.24	0.35	0.56
Velocity RMS				
PMHT	0.08	0.22	0.25	0.31
PMHT-t	0.08	0.06	0.07	0.09
PMHT-t ($p = 0.7$)	0.05	0.06	0.07	0.09
Misassociated				
PMHT	0	0.32	0.41	0.51
PMHT-t	0	0	0	0
PMHT-t ($p = 0.7$)	0	0	0	0

significant timing error, the standard PMHT had much more difficulty following the target manoeuvre. This is evident both in the RMS error and the number of misassociations. In contrast, the PMHT-t managed to track the target through the manoeuver throughout all the trials.

Interestingly, in the case where $p = 1$, the PMHT-t with an incorrectly assumed $p' = 0.7$ was found to have a lower RMS error on the turning scenario. In this case, the algorithm has a mechanism for dealing with outlier measurements that reduces their impact on the tracking performance. When there was non-zero timing error, the mismatched PMHT-t gave almost identical performance to the matched PMHT-t. This indicates that the algorithm is robust to errors in the timing measurement function. It appears that the precise shape of the assumed time measurement pmf is not highly important provided that it gives support over the region where the true function has significant mass.

5 Conclusion

This paper has introduced a method of using PMHT to perform target tracking for the situation where the temporal information is noisy or unreliable. The key idea is to use the PMHT to associate measurements to time instants as well as to targets by treating both assignments as missing data.

The PMHT-t association algorithm determines probabilities for each pairing of measurement and target at each possible time and then estimates the target states by taking the expectation over these assignments.

Simulation experiments were used to demonstrate the effectiveness of PMHT-t to track with measurements with time uncertainty. This method was demonstrated with simulations and improved performance was observed compared with the standard PMHT.

References

[1] Z. Li and H. Leung, "An expectation maximisation based simultaneous registration and fusion algorithm for radar networks," in *IEEE Canadian Conference on Electrical and Computer Engineering*, Ottawa, Canada, May 2006, pp. 31–35.

[2] A. Dempster, N. Laird, and D. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *Journal of the Royal Statistics Society*, pp. 1–38, 1977.

[3] M. Morelande, "Linear filtering with timing uncertainty," in *Proceedings of the 11th International Conference on Information Fusion*, 2008.

[4] U. Orguner and F. Gustafsson, "Target Tracking Using Delayed Measurements with Implicit Constraints," in *Proceedings of the 11th International Conference on Information Fusion*, 2008.

[5] ——, "Distributed Target Tracking with Propagation Delayed Measurements," in *Proceedings of the 12th International Conference on Information Fusion*, 2009.

[6] R. Streit and T. Luginbuhl, "Probabilistic multi-hypothesis tracking," NUWC, Newport, RI, Technical Report 10428, Feb. 1995.

[7] S. J. Davey, *Extensions to the probabilistic multi-hypothesis tracker for improved data association*. PhD dissertation, School Elect. Electron. Eng., Univ. Adelaide, Australia, 2003.

[8] M. L. Krieg and D. A. Gray, "Multi-sensor probabilistic multi-hypothesis tracking using dissimilar sensors," in *Proceedings of the Conference on Acquisition, Tracking and Pointing XI, SPIE Symposium on Aerosense*, vol. SPIE 2086.

[9] S. J. Davey, "Simultaneous localization and map building using the probabilistic multi-hypothesis tracker," *IEEE Transactions on Robotics*, vol. 23, pp. 271–280, 2007.